Real-time Constraints for Adaptive Digital Filters on SISO Stochastic Systems

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Abstract. A Real-time Adaptive Digital Filter (RTADF) interacts with dynamic process with non stationary output answer (in probability sense), having this filter two basic properties: response in quality [13] and response in time [24]: The filter has the critical dynamical restrictions knowing these as synchrony in each evolutionary interval (where the length of each interval is bounded), such that the time filter answer is bonded by each velocity moment, i.e., the filter adjust the time response with respect to the time evolution system, without lose time and information. In additional form but in the same way, the output filter answer has in each iteration an upper and lower limits: bounded these by measurable dynamical intervals [1], considering the properties exposed in [9], [10], [12], [13], [14] with respect to [6], [10], [22] and [27], such that the set of filter answers is bounded too with respect to Distribution Function predefined by the output system answers set, previously characterized. This basic property in probability sense is required for to construct a RTADF. Finally, this paper is organized considering the basic properties about the Real-time and Adaptive Digital Filter, exposing the basic ideas about the Real-time Adaptive Digital Filtering, developing these in the next section describing the RTADF in theoretical form with description in local and global constraints. The RTADF implementation: considering a D. C. motor, developing an extensive analysis of concurrent tasks: precedence constraints and times into a PC are considered as a part of the adaptive filter.

Keywords: *Adaptive Digital Filter, Synchronization, Real-time, Task, Interval.*

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1 Introduction

The adaptive filter is required when neither the fixed specifications are known nor specifications can not be satisfied by time-invariant filters [10], [12], [21] [22] and [26]. Adaptive digital filter is a nonlinear digital filter since its characteristics are dependent on the input signal and consequently the homogeneity additively conditions are not satisfied [21], [22] and [26]. Generally, adaptive Digital Filters are applied in industrial processes into hardware that fulfillment the monitoring & control tasks (to see: [5], [7] & [12]). Adaptive Digital Filters have basic properties: response quality and b) response in time. In other case, the filter description or forecasting filter can be crashed in the real system where it interacts. These properties must be specified and implemented in a *Real-time Adaptive Digital Filters (RTADF)*. A RTADF can be implemented into embedded systems [4] and [12] using digital systems (micro controllers, DSP's, etc) and in digital computers with Real-time Operating Systems (RTOS) [18], [19] & [20].

2 Adaptive Digital Filters

Adaptive filter concept, is commonly used to describe a physic device characteristic, using the information given by the device (to see: [9] and [13]); such that, the filter is applied to a set of data generated by a corrupted signal (the desired signal is a "tricky task" and it is proposed, considering that it is bounded by the four first probability moments obtained by the corrupted signal, having both signal (corrupted and desired) the same distribution function). To talk about the information, it means to consider a lot of states of the system combined with a lot of perturbations (these perturbations have two sources around to the system: *a)* External and *b)* Internal. The noises or perturbations may arise from a variety of sources). The adaptive filter process is used on: a) Monitoring states (issued from the system), and b) Forecast or identify states without noises (originated from emitter, receptor or environment of the system). The adaptive filter theory has a basic area, which describe the main properties about the essential system characteristics (inside and outside qualities of it) [13], [21]: *Identification.* In [21] and [22], was considered two areas required for describing the inner dynamical (knowing as *Estimation*) and basic states; but in present time the estimate operation is used with optimal techniques to adjust the gains into identification filter, this step is knowing as feedback law (to see: [12] and [13]).

The adaptive filters use the information of the system; Error functional $J(k)_i$ (with respect to desired signal) must be converge to minimal value in finite time [6], [12], [13], [21] and [26]; the convergence criterion is defined by a difference between the filtering signal and desired signal and applying on this, the second probability moment [12], [13], [21], and [26], the criterion selected using the error functional, generally is part of the feedback law using to modify the gains into the basic filter or regular the input signal in the filter [7], [12], [13], [21], [22], and [26].

3 Real-time Digital Systems (RTDS)

A Real-time Digital Systems (RTDS) (to see: [4], [8], [11], [14], [20], [28]) could be described as *a digital system that suffice three conditions: a) interaction with physical world, b) correct responses, time constraints from physical world. RTDS* must be synchronized with physical world and this velocity is relative, depending of dynamics of real system and the adaptive filter algorithm, and the properties of the computing system. Then, *RTDS* may be fast or slow depending of real system dynamics; this obeys the criterions exposed in [24]. In a PC, the whole of all activities are processed by *Real-time tasks* [4], [18], [19], [20], [21].

4 Real-time Adaptive Digital Filters (RADF) - Introduction

In [7] was used a Real-time Adaptive Digital Filters implanted in *RTDS* considering the high velocity computers and facility to express the filter in recursive form. In [12] a FIR (*Finite Impulse Response*) Digital Filters was used as a basic "applications to Real-time processing of standard signals". In [25] that the implementation of Digital Filter into PC, was through to consider the times measure of all tasks around the implementation of it into PC (considering the measurable: The A/D and D/A converters, processor operations, filter algorithm and precedence constraints). But the concepts with respect to Real-time Adaptive Digital Filters weren't developed: Kalman Filter was studied in [18], but in it didn't justify the Real-time constraints.

5 Real-time Adaptive Digital Filter (RTADF)

Definition 1 (Real-Time Adaptive Digital Filter). A RTADF is an Adaptive Digital Filter with time constraints imposed by the dynamical process [24] and has the RTDF properties [23]. Including the main properties of it, the RTADF is described as:

Receiving and giving input and output responses, respectively, in synchronized form with respect to dynamics of the process. Inputs and outputs RTADF will be expressed in symbolic form as $\{u(k)\}_g$ and $\{y(k)\}_i$, with $g,i,k \in N^+$, [1][*]. Giving correct responses set with respect to dynamical process: The quality response is defined in local and global senses considered in [12], [13], and [27] and must guarantee stable conditions defined in [2], [6], [9] and [13] and Express the RTADF in recursive form considering the concepts exposed in [3], [7] and [23], guarantee a minimal use of resources and memory, simulating the dynamics of the real process. The feedback law, bounded in time [24] and the quality of its answer [12], [13] and [22], observing the previous points described in this definition previously. The filter and the feedback law into the filter are described in convolution sense in discrete time and in recursive

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* g, i, *represent a probably number of inputs and outputs respect in a concurrent system that evolution to k intervals.*

form, where the set of the changes and interactions between its tasks or instances are bounded, local and globally [12], [13], and [22].

6 RTADF's Constraints into a Digital Computer

When a RTADF in agreement to [23] is implemented in a computer with one processor, its different components use *concurrent Real-time tasks* (to see: [5] and [18]). The real-time task characteristics will be expressed in the following:

Definition 2 (RTDF: *local constraints***).** The RTADF tasks have a lot of local constraints, imposed these by a lot of dynamical characteristics [11], [12]. [18], [19], [20] and [24], with respect to real process:

a) Arrival time. ($l(k)_i$) [5]: Is the time when task becomes ready for execution. $l(k)_i > 0 \forall k \in N^+$., b) Computation time ($C(k)_i$): Is the total time in a instance (to see: [5]) of a task obtained by union of times used for atomic activities., c) Deadline minimum relative ($D(k)_{i\_min}$): Is the minimum time in which a task should be finished[2]. It is a function of $l(k)_i$., d) Deadline Maximum relative ($D(k)_{i\_max}$): Is the maximum time which a task should be accomplished[3], bounded it by the right side with sample time. This deadline is a function of time $l(k)_i$, and depend of the system evolution. e) Start time ($s(k)_i$): Is a time when the task starts with execution. This time depend of: Resource availability, latency times, context change, size of ready queue, etc. Start time is considered a stochastic variable [10] with a range named jitter,[18], and fulfill the condition $s(k)_i < D(k)_{i\_max}$., f) Finishing Time (f(k)$_i$): Is the time when the task execution finished under interval time k. In mathematical sense $f(k)_i \in \lfloor ld(k)_{i\_min}, LD(k)_{i\_max} \rangle$, with $ld(k)_{i\_min} := l(k)_i + D(k)_{i\_min}$, and $lD(k)_{i\_max} := l(k)_i + D(k)_{i\_max}$ [4].,g) Lateness ($L(k)_i$): Is the loss time between finishing time and final interval time k. This time is $L(k)_i := \left| LD(k)_{i\_max} - f(k)_i \right|$., h) Premature time ($P(k)_i$): Is the gain in time generated by finishes task before maximum deadline. This time is defined by $P(k)_i := \left| ld(k)_{i\_min} - f(k)_i \right|$, where $f(k)_i \geq LD(k)_{i\_min}$, generate a premature task answer., i) Sampling Period or Interaction time ($T(k)_i$): It is obtained by Sampling Criteria described in [24]. The RTADF evolution is bounded by Sample Period considering in each evolution the

[2] If the response is obtained before this lower deadline, it means that the system has a null task. In the other hand, if the time is higher that dynamic interval defined by dynamic system period time.

[3] If response is obtained after this deadline, it is bad.

[4] If we sum $l(k)_i$ to $D(k)_{i\_min}$, $D(k)_{i\_max}$, obtain absolute deadlines ($ld(k)_{i\_min}$, $LD(k)_{i\_max}$) [Liu00].

convolution between the filter properly speaking and the feedback law. The main characteristics of this are: a. $T(k)_i := 1/f(k)_{i\_sampling}$. with $T(0)_i = T$, b. $\forall k \exists_i t(k)_i \cap t(k+1)_i - t(k)_i = T(k)_i$ and, c. $\mu\lfloor l(k)_i, LD(k)_{i\_max}) := T(k) + \gamma_i$ where μ is a measurable function in the measure theory sense described in [1], [6] and [27], and γ_i, represent a tiny time (jitter). If $\lfloor l(k)_i, LD(k)_{i\_max})$ is empty, i.e., $\mu\lfloor l(k)_i, LD(k)_{i\_max}) = 0$ at the sense described in [1].

Definition 3 (RTDF: global constraints). The RTADF whole tasks have a global deadline, considering that the infimum value in agreement criteria described in [12], [13], [19] and [22] could be closed with respect to supremum value allowed by dynamical process, and it is the equilibrium point, where the difference between the desired and the filtered signal tend to zero.

7 TRADF: Global performance

In this section will be described the RTADF global properties considering into the feedback law the convergence functional $\{J(m)\}$ when in it the infimum value tend to ε with $m > 0$. The number m represent the interval when the RTADF converge, and ($m_i \uparrow m$, with $i = \overline{1,n}$). The velocity converge depends the gains used into the feedback law considering that the values of $\{J(m)\}$ is lees than one.

Definition 4 (*Convergence: time* t_{i_c}). The time at which the RTADF converges, is $t_{i\_c} := f_i(k = m)$, where m is number of the RTADF convergence interval and t_{i_c} has the condition $d_{i\_c\_min} \le t_{i\_c} < d_i$.

When d_i is convergence deadline, where $d_{i\_c\_min}$ is defined as a minimal convergence deadline imposed by physical world. Guarantying with a feedback law a response on time and synchronized with physical world. The shortest minimal convergence time ($d_{i\_c\_min}$) is bounded with respect to the relative minimal deadline ($D(k)_{i\_min}$), either.

THEOREM 1. The convergence function in probability sense [1], [2],[3], [6] and [27] defined by J_m in [6], [12], [13], [21], [22], [23] and [27], has a value ε_i semi-positive defined, with respect to convergence time described in symbolic form by t_{i_c}.

PROOF. By contradiction, suppose that ε_i is lower than zero$(\varepsilon_i < 0)$. The convergence error, defined in probability sense, is described by the second moment (see for example: [1], [2],[3], [6], [12], [13], [21], [22], [23] and [27]) i.e., , $M\{\Delta_t \Delta_t^T\} \geq \rho_t$ with ρ_t positive semi defined, M represent the mathematical expectation operator and Δ_t is defined as difference between filtered value and real value (to see: [1] and [6]). Now, considering that the $\lim\left(M\{\Delta_t \Delta_t^T\}_{t \to d_i}\right) \to \varepsilon_i$, such that the superior limit of ρ_t when $t \to t_{i\_c}$, is bounded by ε_i, and the inferior limit of ρ_t when $t \to t_{i\_c}$, is bounded by zero. Then, $\varepsilon_i \geq 0$. □

8 RTADF Local behavior (for TVS)

All RTADF are stable (to see: [10], [13], and [15]) if the parameters gains are bounded into the disks [2] defined in the unitary circle for all k ([6], [17], and [27]): $\{a_e(k)\}_i \leq 1, e = \overline{1,n}$.

The optimal parameters obtained with respect the functional J_m and optimal criteria represent symbolically the proper values of modeled system [15] with desired digital signal answer, and these values are stable in discrete sense [2] and [6], without loss the disks into the unitary disk [6], [15], and [17]. Outside of this unitary disk the filter response is unstable, and the filter feedback law isn't convenient, [6] and [13].

THEOREM 2 (Relative maximal deadline *D(k)*$_{i\_max}$). A RTADF fulfill $2f_{i\_max}\left(D(k)_{i\_max} - D(k)_{i\_min}\right) < 1$, f_{i_max} is the dynamical process maximal frequency.

PROOF. Considering [24], the follows condition is true $f(k)_{i\_muestreo} \geq 2f(k)_{i\_max}$, $2T(k)f(k)_{i\_max} \leq 1$, where the $f(k)_{i\_max}$ is the time the filter has convolution the filter with feedback law using in it the functional $J(k)_{i\_max}$. And also, considering to Definition 2 section i, subsections a, b and c; the relative deadline differences is lower than sample time $T(k)_i > D(k)_{i\_max} - D(k)_{i\_min}$ and using transitivity in previous inequalities, we obtain the inequality $2f_{i\_max}\left(D(k)_{i\_max} - D(k)_{i\_min}\right) < 1$. □

9 RTADF Computational times and Deadlines

A RTADF that is implanted in a digital computer with a one processor, the whole of the tasks around of filter will be scheduled in concurrent form; then the total time of RTADF will be conformed by the sum of computational times of total of tasks $C(k)_i = C(k)_X + C(k)_y + C(k)_{a \to X} + C(k)_J + C(k)_{au} + C(k)_{ay} + C(k)_{aye} + C(k)_{fb} + C(k)_{fb \to a}$, where: $C(k)_X$: Computation time of state equation algorithm., $C(k)_y$: Computation time of observable signal equation algorithm (desired signal)., $C(k)_{a \to X}$: Computation time of identification algorithm., $C(k)_J$: Computation time of convergence error equation algorithm., $C(k)_{fb\to a}$: Feedback law, $C(k)_{au}$: Computation time of A/D conversion of input $u(t)$., $C(k)_{ay}$: Computation time of A/D conversion of output y(t)., $C(k)_{ayr}$ $C(k)_{aye}$: Computation time of D/A conversion of the identification output.

THEOREM 3 (Computation time & deadline). A soft FDATR, the computational time $C(k)_i$ will be bounded by deadline $C(k)_i < D(k)_{i\_max}$.

PROOF. a) Suppose the case with the following temporal constraints: $s(k)_i = l(k)_i$, $D(k)_{i\_min} = 0$, $C(k)_i > D(k)_{i\_max}$. $L(k)_i > 0$, $D(k)_{i\_max} \le T(k)_i$. with the task time is described as (to see [20]): $C(k)_i = D(k)_{i\_max} + L(k)_i$. Now, if $L(k)_i$ is expressed as submultiples of $D(k)_{i\_max}$: $L(k)_i = D(k)_{i\_max} r^{-1}$,where $r \in N^+ r > 1$.Then, the equality is: $C(k)_i = (1+\gamma)D(k)_{i\_max}$, with $\gamma := r^{-1}$. The minimal value respect to relative deadline ($D(k)_{i\_max}$), is $r = 1$, then this result implies that the starting hypothesis isn't comply. In accord to Definition 2, we have : $C(k)_i = f(k)_i - s(k)_i$, and $f(k)_i \in \lfloor l(k)_i + D(k)_{i\_min}, l(k)_i + D(k)_{i\_max})$.Considering the worst case: $f(k)_i \to l(k)_i + D(k)_{i\_max}$, and substituting in previous results we obtain: $l(k)_i - s(k)_i < 0$, such that this inequality is true, then the theorem is true. □

Corollary 1: In agreement to [19] about processor assignment, a hard RTADF is formed by a tasks whole such that finished before theirs local deadlines $\{D(k)^j_{i\_max}\}$, and global deadline $D(k)_{i\_max}$, where $D(k)^j_{i\_max} \uparrow D(k)_{i\_max}$, with $j = \overline{1,n}$.

Suppose that the global deadline obey the follows condition: $\exists i$ such as $C(k)_i > D(k)_{i\_max}$. The assignment processor resources in accord to [19] are less to one, and then could be considered: $C(k)_i \left(D(k)_{i\_max}\right)^{-1} \le 1$ this represents a basic contradiction because $C(k)_i < D(k)_{i\_max}$ in agreement to Theorem 3, then this corollary is true.

THEOREM 4 (Synchronization). The RTADF whole responses would be synchronized with respect to temporal properties with respect to dynamical process, avoiding accumulative delay.

PROOF. The finished times set $\{f(k)_i\}$ would be bounded by a intervals set $\{ld(h)_{i\_min}, LD(h)_{i\_max}\}$ for all i,h and $k \in N^+$, such as for all $f(k)_i$, $f(k)_i \ge ld(h)_{i\_min}$ and, $f(k)_i < LD(h)_{i\_max}$. If and only if $h \equiv k$, and this equality implies that RTADF would be synchronized with dynamical properties of physical process.

By the other hand if: a) If $h>k$ then, the RTADF generate in it, accumulative delays and avoid synchronization. b) If $h<k$ then the RTADF generate accumulative advances and avoid synchronization. In the last two points, the RTADF tasks whole, don't have synchronization with physical process. The feedback law doesn't synchronized with the evolution of the input signal and state of the digital filter, then the results are bad because the answer of RTADF is obtained with delayed gains.□

10 RTADF Implementation Example in a Digital Computer

We will implement a RTADF using this with simple basic feedback law modifying inversely the gains of parameter evolution applied to D.C. motor.

To implement a RTADF into PC required a tool set:
a)D.C. Motor, 20 V, 1 A, 1800 rpm, two poles, permanent field., b) Power unit A/D, D/A, 5 V input, 20 V output, 0.05 A input, 5 A output., c) PC Pentium III 400 MHz, 64 MB RAM., d) ADC card PCL 818L, e) QNX® 4.24 Real-time Operating system., f) MicroPhoton Development Kit®.

11 RTADF Characteristics

The characteristics of model and RTADF are in agreement to [21]: a) SISO model (i=1), linear, nonstationary model in agreement to [2], [6], and [13],., b) Step input of 20 volts., c) Wiener Algorithm [27]., d) Noises v(k) and w(k) are correlated with observable signal described by y(k) but they aren't correlated between them., e) The dynamical model will be described as: $x(k+1)=a(k)*x(k)+v(k)$, $y(k)=x(k)+\omega(k)$, where: $x(k+1)$ is the system state, $y(k)$ is the observable signal (angular velocity), $v(k)$ and $\omega(k)$ are internal noise and external noise respectively and, "$a(k)$" is the parameter.
The identificator in basic sense is described as $\hat{y}(k)=\hat{a}(k)*y(k)+sing(\hat{y}(k)-y(k))*J(k)$, Using the gradient as an optimal tool the estimator with feedback is expressed: $\hat{a}(k):=b(k)^{-1}p(k)+sing(\hat{a}(k)-a)*J(k)$, where, in recursive form , P(k)= P(k-1)+ y(k) y(k-1), B(k)= B(k-1) + y(k-1) y(k-1).
The functional defined in [6] and [13] is expressed as: J(k)=E ($\Delta(k)\ \Delta^T(k)$), with $\Delta(k):=\hat{a}(k)-a$.
In the model described by (17) and (18), will be considering the next dates for it implementation [21]: a= 0.35, σ(v(k))= 0.95, σ(w(k))= 0.99.

12 RTADF implementation

The scheme tasks could be seeing in [4], [8] and [16]. Experimental implementation of FDTR is made by concurrent tasks: a) Maximal relative deadline $D(k)_{max}$, is equal to period T(k)., b) Bounded starting time (s_p-l(k) ∀k) was obtained by first probability moment, such as s_p = l(k)+0.0015 ms for each task., c) D.C. motor sampling period T(k) is 20 ms., d) Minimal relative deadline is $D(k)_{min}$= 2.5 ms., if the deadline is lower than 2.5 ms, the motor has bad operation., e) Convergence deadline is: d= 3 s. The RTADF require creating next task: Xk: System states algorithm, C_{Xk}= 0.237 ms, Yk: Observable signal algorithm, C_{Yk}= 0.289 ms, Bk: Observable signal variance, C_{Bk}= 0.268 ms, Pk: Ricatti equation algorithm, C_{Pk}= 0.249 ms, Ak: Parameter estimator algorithm, C_{Ak}= 3.252 ms, Jk: Error functional, C_{Jk}= 0.245 ms, FB: the feedback $C(k)_{fb\to a}$ =0.21ms, Au: A/D conversion of input, C_{Au}= 0.310 ms, Ay: A/D conversion of observable signal, C_{Ay}= 0.302 ms, Aye: D/A conversion of estimate signal, C_{Aye}= 0.314 ms, O: Parent task, C_O = 0.261 ms., C(k) is: 5.7305 ms. The computational times is described as $C(k)= C_{Xk} + C_{Yk} + C_{Bk} + C_{Pk} + C_{Ak} + C_{Jk} + C_{Au} + C_{Ay} + C_{Aye} + C(k)_{fb\to a}$,. And the times obtained: T(k)= 20 ms, l(k)= T(k-1)k ms, s(k)= l(k)+ 0.0135 ms, C(k)= 5.7305 ms, $D(k)_{min}$= 2.5 ms, $D(k)_{max}$= 20 ms, f(k)= 5.7305 ms, L(k)= 14.269 ms, P(k)= 3.2305 ms. The convergence time proved experimentally is: m= 113 intervals, t_c=2.45 s, d= 3 s, t_c<d is complied. The time value is 0.55 (To see: [21]).

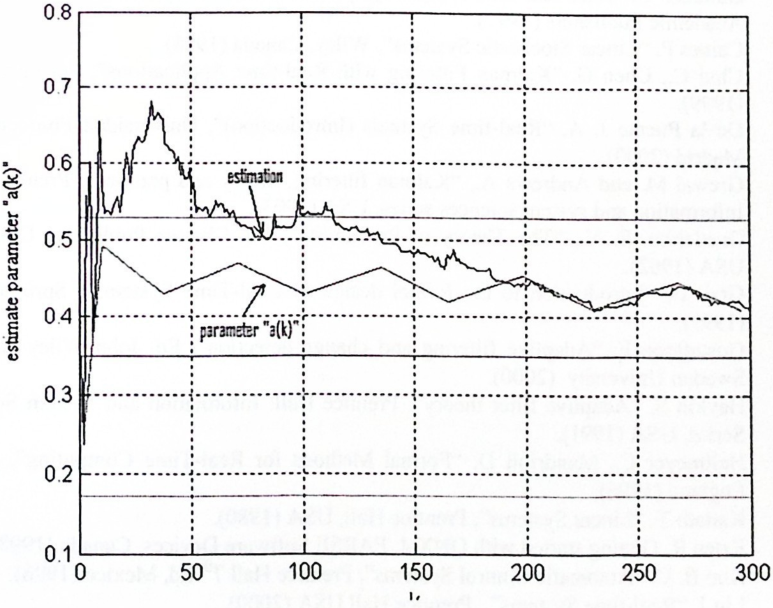

Fig. 1. RTADF: parameter estimator.

13 Conclusions

A Real-time Adaptive Digital Filter (RTADF) was described as a system that interacts with dynamic processes with non stationary output(s) (in probability sense), and has it two basic properties: response in quality [13] and in time [24]. The critical problem is the synchrony of it with respect to evolutionary answer into dynamic intervals.
In this paper we describe in theoretical sense the basic description about the RATDF, considering as a basic property the distribution Function about the output system answer, previously defined.
In practical sense we use the RTADF as estimator, and the illustration depicted, describe in illustrative form the convergence between the estimator and real parameter, with out the time properties that this filter require to accomplish.

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